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GENERATION OF GEOMAGNETIC OSCILLATIONS IN THE LATE STAGE OF A CAMOUFLET EXPLOSION

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Generation of geomagnetic disturbances is studied at the late stage of an underground nuclear explosion, where the originally displaced geomagnetic field penetrates into the explosion cavity again. The strength of the electric field of a geomagnetic disturbance due to oscillations of the magnetic moment is calculated. A method for analyzing records of geomagnetic disturbances is developed that permits one to take into account the character of quasiperiodic oscillations in the signal. The calculations are compared to results of the experiments performed in the state of Nevada in 1958.

All the papers devoted to examination of geomagnetic disturbances from underground nuclear explosions (see, e.g., [1–4]) have been focused on the main portion of the signal, which is a two-polar pulse with a rather short positive phase and a lengthy, asymptotically vanishing, negative phase. The ratio of the durations of these phases is about 1 : 10. The experimental curves from [1, 5] suggest that many geomagnetic disturbances are terminated by a quasiperiodic process. The amplitude of these oscillations is much smaller than the amplitude of the initial phase of the signal, but in many experimental records, oscillations can be distinguished with confidence against the noise background. In the present paper, the processes occurring in the explosion cavity in the late stages of explosion are considered and the dependence of the frequency period of geomagnetic disturbances at the pulse "tail" on the explosion characteristics is established.

In practice, two cases of explosion occur, which are considered, for example, by Chedwick et al. [6] and Brode [7].

1. Before an explosion, the charge is tightly surrounded by rock [6]. During the explosion, the rock evaporates, and a shock wave propagates and damps in ground. A cavity forms around the charge, and the walls of the cavity move, producing a rarefaction wave.

2. An explosion is produced in a chamber of large dimensions (decoupling) to decrease the seismic effect [7] prepared beforehand. The shock wave is locked in the cavity and undergoes multiple reflections from the practically immovable walls of the chamber.

Below, we consider the first case of explosion in a continuous medium.

A physical concept for generation of geomagnetic disturbances from underground nuclear explosions has not yet been developed. In [1], different models for generation of geomagnetic disturbances are discussed, and it is shown that the main mechanisms involved in the formation of low-frequency signals are displacement of the geomagnetic field from the hot expanded plasma ball and appearance of a magnetic dipole moment of the ball. Despite the great number of works in this field, the problem of generation of electromagnetic fields by expansion of a plasma ball with high electrical conductivity in a conducting magnetized nonuniform medium (ground) in the presence of the ground–air boundary has not been solved rigorously. Researchers who attempted to solve this problem using various simplifications did not obtain definite relations between

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the time characteristics of disturbances and the source parameters. Therefore, we chose another approach to developing a physical concept of the phenomenon.

Our concept is based on the assumption that the plasma formed in explosion has a time-varying magnetic dipole moment $M(t)$. Let, at the moment of explosion, there be evaporation of the rock surrounding the nuclear charge. The hot, completely ionized gas expands, separating the chamber walls. At the initial stage of the expansion, where the temperature and electrical conductivity of the plasma are very high, the magnetic field is displaced from the site of explosion. The ring currents originating on the surface of the fire ball produce a magnetic moment, which rapidly increases with expansion. A rarefaction wave propagates into the plasma from the chamber walls, which move apart. The wave is cumulated at the center of the spherical cavity, reflected from it, and travels to the walls again. This process occurs many times, and the wave speed depends on the gas pressure and density. At the early stages, the radial motion of the plasma proceeds in a wave region that is free from the magnetic field, and, hence, this does not lead to modulation of the magnetic moment of the plasma ball. At the late stages, where the expansion slows down, the ball cools and reverse diffusion of the geomagnetic field into the ball begins (or, because of the plasma instability, the plasma formation breaks up). These processes lead to a gradual decrease in the magnetic moment, and the rate of the decrease depends on the mechanisms whereby the geomagnetic field penetrates into the cavity. By this time, the rate of expansion of the cavity is much lower than the velocity of sound, and the process of expansion can be considered quasistatic. A standing sound wave is established in the spherical cavity. The velocity direction of the gas particles in the wave is periodically changed, and this gives rise to alternating oscillating currents in the plasma volume and oscillations of the magnetic moment.

Thus, there are two mechanisms of generation of geomagnetic disturbances, and they operate at different stages of explosion. Initially, there is a sudden burst of radiation due to the rapid displacement of the magnetic field from the explosion region, and then the burst relaxes because of the cooling of the plasma. The splash is followed by more or less regular oscillations of the magnetic moment of the plasma in its radial motion caused by the standing wave. The initial stage of the explosion is examined in [1–4]. In the present paper, we examine only the late stage of the explosion.

Let us consider, for simplicity, a vertical geomagnetic field. We introduce a spherical coordinate system where the Z axis is directed along the geomagnetic field and the coordinate origin is at the center of the explosion.

We calculate the magnetic moment of the plasma in the explosion cavity in the late stage. Let the walls of the cavity expand at the late stage of explosion by the law

$$u(t) = \frac{dR}{dt} = V_1 \exp\Big(-\frac{t-t_1}{\tau}\Big), \tag{1}$$

where τ is the characteristic time of expansion of the cavity, V_1 is the rate of expansion at time $t = t_1$, from which approximation (1) is considered valid, and R is the radius of the cavity. We begin timing with $t = t_1$, by setting $t_1 = 0$.

Since the velocity of sound a in the gas is much higher than the velocity of the cavity walls, a standing wave arises in the cavity before the dimensions of the cavity are significantly changed ($a \gg dR/dt$). The wave equation for the velocity potential $\boldsymbol{V} = \nabla\varphi$ has the form $\partial^2\varphi/\partial t^2 = a^2\Delta\varphi$. We seek its solution in the form of a monochromatic wave:

$$\varphi = \frac{A}{r}\sin(kr)\exp(-i\omega t). \tag{2}$$

Here $k = \omega/a$. Differentiating (2) with respect to r, we find the gas velocity in the wave propagating in the cavity:

$$V = A\exp(-i\omega t)\Big[\frac{k\cos(kr)}{r} - \frac{\sin(kr)}{r^2}\Big]. \tag{3}$$

The gas velocity at the cavity wall is equal to the velocity of the wall (1). We represent it as the Fourier integral

$$\frac{dR}{dt} = u(t) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} u_\omega \exp(-i\omega t)\, d\omega,$$

where

$$u_\omega = \int\limits_{-\infty}^{\infty} u(t) \exp(i\omega t)\, dt = \int\limits_{0}^{\infty} V_1 \exp(i\omega t - t/\tau)\, dt = \frac{-V_1}{i\omega - 1/\tau}.$$

The constant A is determined from the condition that for each monochromatic component, the gas velocity (3) at $r = R(t) \approx R_0$ is equal to $u_\omega \exp(-i\omega t)$. Then,

$$A\Big[\frac{k\cos(kR_0)}{R_0} - \frac{\sin(kR_0)}{R_0^2}\Big] = -\frac{V_1}{-1/\tau + i\omega},$$

whence

$$A = -\frac{V_1}{-1/\tau + i\omega} \frac{R_0^2}{kR_0 \cos(kR_0) - \sin(kR_0)}, \tag{4}$$

where R_0 is the maximum radius of the cavity.

Using (3) and (4), by inverse Fourier transformation we obtain the gas velocity in the cavity related to the motion of the wall at the final stage of the expansion:

$$V(t,r) = -V_1 \frac{R_0^2}{r^2} \int\limits_{-\infty}^{\infty} \frac{\exp(-i\omega t)[kr\cos(kr) - \sin(kr)]}{(i\omega - 1/\tau)[kR_0\cos(kR_0) - \sin(kR_0)]} \frac{d\omega}{2\pi}. \tag{5}$$

We calculate integral (5) using the calculus of residues. The denominator of integrand (5) vanishes for $\omega = 1/(i\tau)$ and $\tan(kR_0) = kR_0$. The minimum root of the last equation is $k_0R_0 \approx 4.5$. Hence, one value of the frequency is $\omega_1 = 4.5a/R_0 = \omega_0$ and the other (negative) value is $\omega_2 = -4.5a/R_0 = -\omega_0$. The influence of overtones is ignored. In this case, from (5) subject to the condition $\omega_0\tau = 4.5a\tau/R_0 \gg 1$, we obtain

$$V(t,r) = -V_1 \frac{R_0^2}{r^2} \frac{r/(a\tau)\cosh(r/(a\tau)) - \sinh(r/(a\tau))}{R_0/(a\tau)\cosh(R_0/(a\tau)) - \sinh(R_0/(a\tau))} \exp(-t/\tau)$$
$$+2V_1 \frac{k_0r\cos(k_0r) - \sin(k_0r)}{k_0^2r^2 \sin(k_0R_0)} \cos(\omega_0 t). \tag{6}$$

For $t > \tau$, expression (6) contains only the second term, which describes a standing wave with the fundamental frequency. Because of the dissipative processes, the standing wave damps with time. The damping is taken into account by introducing the coefficient β that satisfies the inequality $\beta \ll 1/\tau$, so that after a lapse of time $t = \tau$, the gas oscillations in the standing wave continue. We take the moment $t_2 > \tau$ as the reference time. Then, we have

$$V(t,r) = 2V_1 \frac{k_0r\cos(k_0r) - \sin(k_0r)}{k_0^2r^2\sin(k_0R_0)} \exp(-\beta t)\sin(\omega_0 t). \tag{7}$$

We find the magnetic moment due to the plasma motion under the action of the standing wave. Let the plasma conductivity be equal to σ_p. The gas is in magnetic field B. Then, the ring electric currents have the form $j_\varphi = \sigma_p BV(r,t)\sin\theta$ and the magnetic moment is

$$M(t) = 2\pi \int\limits_0^{\pi/2} \int\limits_0^{R_0} j_\varphi r^3 \sin^2\theta\, dr\, d\theta$$

$$= 4\pi BV_1 \frac{\exp(-\beta t)}{\sin(k_0R_0)} \sin(\omega_0 t) \int\limits_0^{\pi/2} \sin^3\theta\, d\theta \int\limits_0^{R_0} \frac{\sigma_p r^3}{k_0^2 r^2} [k_0r\cos(k_0r) - \sin(k_0r)]\, dr.$$

We assume that σ_p depends only on time. Then, calculating the integrals and taking into account that $k_0 R_0 = 4.5$ and $\tan(k_0 R_0) = k_0 R_0$, we obtain

$$M(t) = 0.38\sigma_p B V_1 R_0^4 \exp(-\beta t) \sin(\omega_0 t). \tag{8}$$

Expression (8) is valid only for times when the magnetic field B has penetrated into the cavity. Assume, at time $t = t_2$, that the geomagnetic field has penetrated into the cavity. From this moment, the conductivity σ_p is considered constant. The magnetic moment is written as

$$M(t) = \begin{cases} M_0 \exp(-\beta t) \sin(\omega_0 t) & \text{for} \quad t \geqslant 0, \\ 0 & \text{for} \quad t < 0, \end{cases}$$

where $M_0 = 0.38\sigma_p B V_1 R_0^4$. The time t_2 was previously set equal to zero.

The magnetic moment $M(t)$ depends on the velocity V_1 of motion of the spherical wall of the cavity. If there is no motion, magnetic moment is absent since in the mechanism considered only motion of the cavity walls produces an acoustic standing wave. We represent $M(t)$ as a Fourier integral

$$M(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} M_\omega \exp(-i\omega t)\, d\omega,$$

where

$$M_\omega = \int_{-\infty}^{\infty} M(t) \exp(i\omega t)\, dt = M_0 \int_0^{\infty} \exp(-\beta t + i\omega t) \sin(\omega_0 t)\, dt.$$

Integrating and assuming that $\beta \ll \omega_0$, we obtain $M_\omega = M_0\omega_0/(\omega_0^2 - \omega^2 - 2i\beta\omega)$. Let us calculate the electric-field strength of geomagnetic disturbances. Because of the high conductivity of ground and the low frequency (about a hertz and fractions of a hertz) of the disturbances considered, we ignore displacement currents. If the geomagnetic field is vertical, the conductivity currents have a ring shape (the geomagnetic line passing through the point of explosion is the symmetry axis), are located in planes that are coplanar to the planar ground–air interface, and exist only in a conducting medium. From the Maxwell equations $[\nabla \boldsymbol{H}] = \boldsymbol{j}$ and $[\nabla \boldsymbol{E}] = -\partial \boldsymbol{B}/\partial t = -\mu_0 \boldsymbol{H}/\partial t$, we obtain $[\nabla[\nabla \boldsymbol{E}]] = -\mu_0 \partial \boldsymbol{j}/\partial t$ and $\nabla(\nabla \boldsymbol{E}) - \Delta \boldsymbol{E} = -\mu_0 \partial \boldsymbol{j}/\partial t$.

We supplement the Maxwell equations by the Ohm's law $\boldsymbol{j} = \sigma \boldsymbol{E}$, where σ is the conductivity of ground. Taking into account that the currents are closed and there is no separation of electric charges, i.e., $\nabla \boldsymbol{j} = 0$, we obtain the following equation for the current density:

$$\Delta \boldsymbol{j} = \mu_0 \sigma \partial \boldsymbol{j}/\partial t. \tag{9}$$

Equation (9) is a diffusion equation, and, hence, one can speak of diffusion of electric currents in a conducting medium.

In the case of the vertical geomagnetic field, there is only one azimuthal component of the current density j_φ, for which the equation in spherical coordinates has the form

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial j_\varphi}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta} \sin\theta \frac{\partial j_\varphi}{\partial \theta} - \frac{j_\varphi}{r^2 \sin^2\theta} = \mu_0 \sigma \frac{\partial j_\varphi}{\partial t}. \tag{10}$$

Khanakhbei [8] showed that under certain conditions, which will be formulated below, the fields generated in air by a magnetic dipole immersed in ground have little effect on the electric currents in ground. This make it possible to solve the problem of the diffusion of currents in a conducting medium ignoring the dielectric properties of air. According to the aforesaid, we seek a solution in the form

$$j_\varphi = \sin\theta \exp(-i\omega t) f(r). \tag{11}$$

Substituting (11) into (10) and solving Eq. (10), we obtain

$$f(x) = \frac{C}{x}\left(1 + \frac{i^{1/2}}{x}\right) \exp\left(-\frac{x}{i^{1/2}}\right), \qquad x = (\mu_0 \sigma \omega)^{1/2} r, \qquad C = \text{const}. \tag{12}$$

The calculations in [8] show that at $h/r \approx 0.1$ (h is the depth of explosion) and $x \leqslant 1$, the presence of a dielectric half-space can change the field magnitude obtained from formula (12) by not more than 10%. At $x = 1$, the exponent in (12) decreases by a factor of e compared to unity. The influence of the dielectric half-space on the fields in ground increases with increase in x. However, since the function $f(x)$ rapidly decreases, solution (12) can be extended to the entire space of the variable x.

The constant C is related to the magnetic moment by

$$M_\omega = \pi \int_0^{\pi/2} \int_0^{\infty} j_\varphi r^3 \sin^2\theta \, dr \, d\theta = \pi \int_0^{\infty} j(r,\omega) r^3 \, dr \int_0^{\pi/2} \sin^3\theta \, d\theta = \frac{2\pi}{3} \int_0^{\infty} j(r,\omega) r^3 \, dr, \tag{13}$$

where $j(r,\omega) = f(x)$, and integration is performed over the volume in which currents are present. Using (12) and (13), we define the current density as

$$j(r,t) = \frac{M_0\omega_0}{2\pi} \int_{-\infty}^{\infty} \frac{(\mu_0\sigma\omega)^2 \exp(-i\omega t)}{\omega^2 - \omega_0^2 + 2i\beta\omega} \frac{i^{1/2}}{x}\left(1 + \frac{i^{1/2}}{x}\right) \exp(-x/i^{1/2}) \, d\omega. \tag{14}$$

The integrand in expression (14) has two poles of the first order. To calculate the integral, we use the calculus of residues, introduce the parameter $x_0 = (\mu_0\sigma\omega_0 r^2)^{1/2}$, and simplify the expression obtained, by setting $\beta \ll \omega_0$. Then, the electric-field strength of geomagnetic disturbances is

$$E(r,t) = \frac{j(r,t)}{\sigma} = \frac{\sqrt{2}M_0 x_0^3}{4\pi r^4 \sigma} \exp\left(-\beta t - \frac{x_0}{\sqrt{2}}\right)\left[\left(1 + \frac{1}{\sqrt{2}x_0}\right)\cos\left(\omega_0 t - \frac{x_0}{\sqrt{2}}\right) - \sin\left(\omega_0 t - \frac{x_0}{\sqrt{2}}\right)\right]. \tag{15}$$

The electric-field strength (15) has only an azimuthal component and is maximal, according to (11), at angle $\theta = \pi/2$. If the depth of explosion is small compared to the distance at which the electromagnetic-field disturbances due to the explosion are recorded, maximal disturbances must be observed at ground. According to (15), the oscillation amplitude depends on the distance between the site of explosion and the point of observation by the law $j \sim (x_0^3/r^4)\exp(-x_0/\sqrt{2}) \sim r^{-1}\exp[-r(\mu_0\sigma\omega_0)^{1/2}/\sqrt{2}]$ and decreases exponentially with distance.

We estimate the distance at which the oscillation amplitude decreases by a factor of e (i.e., at $x_0 = 2^{1/2}$ or $\mu_0\sigma\omega_0 r^2 = 2$), for the "Gnome" explosion produced in salt beds. We set the conductivity of ground equal to $\sigma \approx 1.7 \cdot 10^{-2}$ C/m [1]. The cyclical oscillation frequency $\omega_0 = 2\pi/T_0$ is taken from [9]. We calculate the wave speed $a = (\gamma p/\rho)^{1/2} = (\gamma R_g T/M)^{1/2}$ (R_g is the universal gas constant, T is the gas temperature, and M is the molar mass of the gas) at the final stage of expansion. We assume that at the final stage the temperature in the cavity is close to the melting point of salt: $T \approx 1200$ K. At the end of expansion of the cavity, the effective adiabatic exponent is $\gamma \approx 1.1$. For the molar mass of NaCl $M = 58.5$ kg/kmole, we obtain $a \approx 440$ m/sec. The maximal radius R_0 of the cavity calculated by comparing the rock pressure with the gas pressure in the cavity [9] is equal 20 m. As a result, we obtain $r \approx 1$ km.

To compare the theoretical results with experimental data, it is necessary to perform a careful independent analysis of experimental records of geomagnetic disturbances to establish correlations between the signal parameters and the known characteristics of the source. The techniques we used to study magnetograms differ radically from the conventional Fourier methods of analyzing curves and considerably decrease the influence of high-frequency noise on the error in determining the required parameters. The material accumulated in a computer data bank using these measuring techniques is subjected to mathematical processing by means of an application package [10–12]. This package is based on methods of nonparametric statistics and can work even with small samples. One method of processing magnetograms that is used to study the phase of a quasiperiodic signal is described below.

The majority of experimental records show oscillation processes that accompany the first burst of the signal. The oscillations are not strictly periodic but small segments of the process can be approximated by a sinusoid without a large error. In studies of such signals, it is convenient to use an approximating model of the form

$$F(t) = \cos(f(t)) + s(t),$$

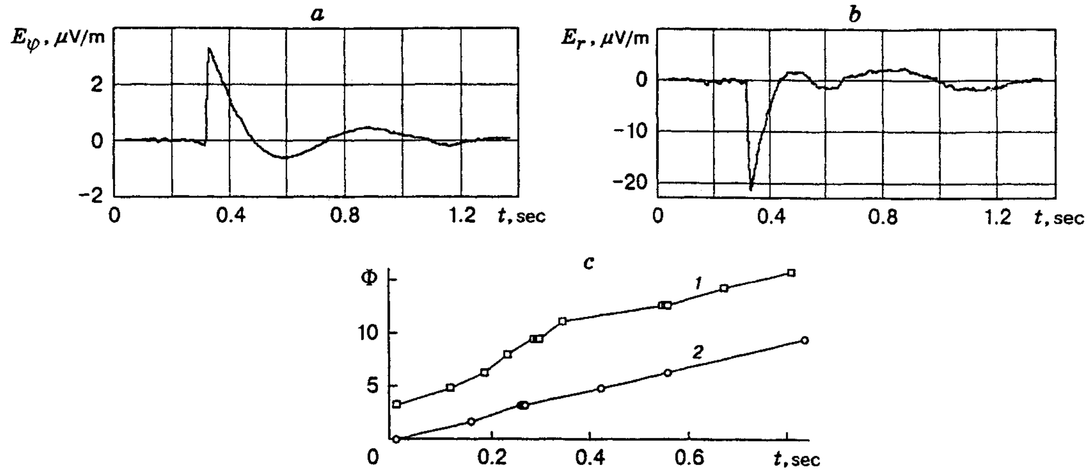

Fig. 1

where $F(t)$ is the examined signal, $f(t)$ is a monotonic function of time, and $s(t)$ is the noise component of the signal. The approximation proposed above reduces investigation of the regular component of the signal to analysis of the function $f(t)$, which will be called the conditional phase of the signal. In the presence of noise of a rather large amplitude, pointwise restoration of the function $f(t)$ is difficult. However, even with intense noise, the neighborhoods of zeroes and extrema of the signal can be obtained rather reliably. Assigning the corresponding conditional values of the sinusoid phase (0, $\pi/2$, π, $3\pi/2, \ldots$) to the neighborhoods of zeroes and extrema, plotting the dependence of this phase on time, and approximating this plot by a smooth curve, one constructs a model for the regular part of the signal in the form $\cos(f(t))$. From comparison of the model curve with the initial signal, it is possible to estimate the quality of the approximation and find the necessary systematic corrections. This procedure was performed with the records of the electrical component of geomagnetic disturbances in [1].

As an example, we give curves of the conditional phase for the "Bilby" explosion (Fig. 1). The record of the "Bilby" explosion contains the azimuthal E_φ (Fig. 1a) and the radial E_r (Fig. 1b) components, which differ in the number of oscillations over the total period of recording the signal (about 1 sec). The first deviation of the azimuthal signal is positive, and that of the radial signal is negligible. Plots of the conditional phase Φ versus time for these two records are given in Fig. 1c. Squares show the values of the phase $\Phi = \pi$, $3\pi/2, \ldots$ for the record of E_r (curve 1), and circles show the values of $\Phi = 0$, $\pi/2$, $\pi, \ldots$ for E_φ (curve 2). All extrema and zeroes of the azimuthal record are on a straight line, which corresponds to oscillations with constant frequency. The rate of growth of the conditional phase from the first minimum on the record of E_r to the nearest zero coincides with the growth rate of the phase in the latest stages of the signal (curve 1 in Fig. 1c). Moreover, this rate is equal to the rate of variation in the phase of the azimuthal signal E_φ (curve 2 in Fig. 1c) since curves 1 and 2 in Fig. 1c are practically parallel. The maxima and minima at the "tails" of both records (Fig. 1a and b) are synchronous. If one assumes that the mechanisms of development of the initial and subsequent stages of the signal are different, the transition from one mechanism to the other in Fig. 1 occurs approximately at 300 m/sec. This procedure of selecting a function to approximate the real signal was employed to determine the "periods" of oscillations in different records at the late stage of geomagnetic disturbances. These measurements were used to establish the relationship among the "period," the energy of explosion, and the depth of location of the charge.

We study the dependence of the oscillation period on the energy of explosion and depth of location of the charge. According to (8), the oscillation period T_0 depends on the speed of acoustic waves a and the maximum radius of the cavity R_c. The radius R_c depends on the energy of explosion, the properties of rock, and the depth of location of the charge. The wave speed is determined by the pressure and density of the evaporated material in the cavity and, hence, on the properties of the rock and the depth of explosion. We use

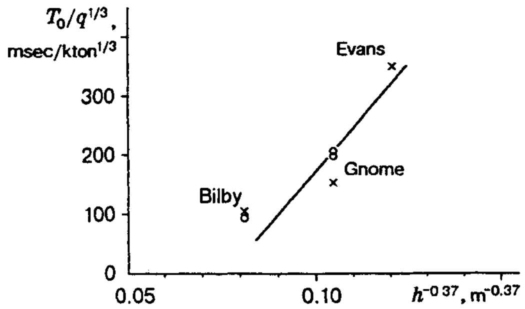

Fig. 2

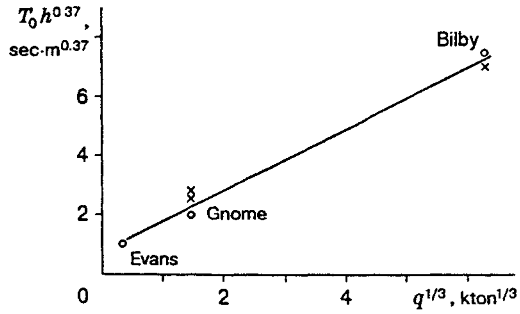

Fig. 3

the calculations of [9], where the state of heated gas in the cavity is studied at various stages of gas expansion. The calculations in [9] were carried out for various natural materials typical of the state of Nevada, where the underground nuclear explosions were conducted. We use records of the geomagnetic disturbances produced by these explosions.

In [9], it is shown that at the moment the formation of an explosion cavity is completed, the pressure p_c in the cavity is directly proportional to the rock pressure at the depth of explosion $p_c = \alpha\rho g h$, where h is the depth of location of the charge, ρ is the rock density averaged over the height h above the charge, and α is a constant of proportionality that depends on the type of rock ($\alpha = 2.0$ for salt and $\alpha = 1.4$ for water-saturated tuff). Next, we assume that the average density of rock is about the same. The final pressure in the cavity exceeds the rock pressure $\rho g h$ since the effective shear strength of the medium causes a counteraction to the enlargement of the cavity. When the cavity walls stop, the volume density of the evaporated material can defined as $\rho_c = \rho_0 (R_v/R_c)^3$, where ρ_0 is the density of the evaporated material at the moment of explosion and R_v is the radius of the cavity formed at the moment the rock evaporates but expansion has not yet begun. Butkovich [9] reports the following values of R_v^{1kt} calculated per 1 kton for various natural materials, in particular, $R_v^{1kt} = 1.83$ m for granite, $R_v^{1kt} = 2.06$ m for water-saturated tuff, and $R_v^{1kt} = 2.25$ m for salt. The final radius of the cavity is $R_c = R_c^{1kt} q^{1/3}$, where q is the TNT equivalent of the explosion (in kilotons) and R_c^{1kt} is the final radius of the cavity for 1 kton. The relation between the R_c^{1kt} and the final pressure p_c in the cavity is obtained by approximating the graphs of $p_c(R_c^{1kt})$ from [9], and it has the form

$$\begin{aligned} R_c^{1kt} &= 1.80 p_c^{-0.235} \quad \text{for salt;} \\ R_c^{1kt} &= 1.51 p_c^{-0.255} \quad \text{for water-saturated tuff.} \end{aligned}$$

Substituting $p_c = \alpha\rho g h$ into these expressions, we obtain the final radius of the cavity and the corresponding oscillation period:

$$\begin{aligned} T_0 &= A_1 q^{1/3} h^{-0.38} \quad \text{for salt;} \\ T_0 &= A_2 q^{1/3} h^{-0.37} \quad \text{for tuff} \end{aligned} \tag{16}$$

(the numerical coefficients are not written here).

To verify dependences (16), we analyzed the records of geomagnetic disturbances from [1, 5], for which both the depth and power of explosions are known. Figure 2 shows the plot of the oscillation period referred to the cube root from the TNT equivalent of explosion versus the depth of explosion to power −0.37. The measurement data are given for the three experiments: "Evans," "Gnome," and "Bilby," for which the TNT equivalents were 0.055, 3.1, and 235 ktons, and the depths were 256, 361, and 714 m, respectively. The points on the plot correspond to the periods evaluated from records of the azimuthal component of the disturbance, and the crosses correspond to the periods calculated from records of the radial component. Figure 3 shows the dependence of the quantity $T_0 h^{0.37}$ on the cube root of the TNT equivalent of explosion in the same experiments. Figures 2 and 3 demonstrate the directly proportional relation between the corresponding parameters, which is in good agreement with formula (16).

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